Contents

1 The Confidence Interval is not the probability

adapted from:

- `https://qr.ae/pNrVlx`
- `https://qr.ae/pNrV6y`

I assume that the motivation for this question is that most statistics books emphasize the fact that, once you have taken a sample and constructed the confidence interval (CI), there is no longer any randomness left in a CI statement (except for the Bayesian point of view which thinks of μ as being a random variable).

That is, when reporting a CI: I am 95% confident that the mean is between 25.1 and 32.6 is correct. There is a 95% probability that the mean is between 25.1 and 32.6 is WRONG. Either μ is in that interval or not; there is no probability associated with it.

1.1 The Reasoning

Suppose that somewhere on the wall is an invisible bullseye a special point (call it μ) which only I can see. Im going to throw a dart at μ . Based on long observation, you know that when I throw a dart at something, 95% of the time, my dart will hit within 6 inches of what I was aiming at. (The other 5% of the time, I miss by more than 6 inches.) When you see where that dart lands, you will draw a circle around it with a radius of 6 inches.

It is correct to say:

> The probability that μ will be in that circle is 95%.

The reason that is correct is, I have not yet thrown the dart, so the location of the circle is random, and in 95% of repetitions of this dart-throwing, circle-drawing routine, μ will be in the resulting circle. Now if I actually take aim, throw my dart, and it hits

$$\text{right here} \quad \Longrightarrow \quad \cdot$$

It is no longer correct to talk about probabilities. You can be pretty sure that μ is within that circle. To be specific, pretty sure = 95% confident. But you cannot say that the probability that μ is in that circle is 95%, because μ is not random.This throw might have been one of the 5% of throws that miss μ by more than 6 inches.

Lets assume we want a 95% CI for μ from a normal population with a known standard deviation σ , so the margin of error is:

$$M = 1.96 \frac{\sigma}{\sqrt{n}}$$

Then $\overline{X}$ is the dart we are throwing at μ.

Before you take the sample and compute the mean, you have:

$$P(\overline{X} - M < \mu < \overline{X} + M) = 95\%$$

This is correct because $\overline{X}$ is a random variable. However, once you compute the mean $\overline{x}$ (lowercase x meaning it is now just a number, not a random quantity), the inequality:

$$\overline{x} - M < \mu < \overline{x} + M$$

is either true or false; the dart has landed at $\overline{x}$, and we dont know if this was one of the throws that is within M of μ.

2 Classical Confidence Interval

A classical confidence interval contains all values for which the data do not reject the null hypothesis that the parameter is equal to that value.

This does not necessarily tell you anything regarding the probability that the parameter is in the interval.

If however you intended to take a sample of the data and draw a 92% confidence interval, there would be a 92% probability of the population mean being within that interval, if however you drew that sample and created that interval the the probability of the invisible point μ being within that interval can't really be known because we just don't know where it is relative to the dart (i.e. how well the sample reflects the population).